Analytical and FE Modelling of Ground Vibration Transmission
Modélisation analytique et EF de la transmission des vibrations sur le sol

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ABSTRACT

Vibrations, especially during the construction phase of new buildings in urban areas, may cause harm to neighbouring structures in terms of increased settlements and cracks and a substantial annoyance to humans in the area. This paper is devoted quantification of footing vibrations and vibrations at the surface of a half-space caused by a known vibration source placed on the surface. The half-space consists of different layers with given elastic and dynamic properties. The problem is solved numerically by means of a dynamic finite element (FE) program for a number of typical suites of layers. Furthermore, algorithms are proposed to predict the vibration pattern for other layer combinations.

RÉSUMÉ

Les vibrations, notamment pendant la construction d'édifices en zone urbaine, peuvent causer un certain nombre de dommages (affaissement aggravé, fissures, etc.) aux structures situées à proximité et des nuisances aux riverains. Cet article concerne la quantification des vibrations de la surface d'un espace semi-infini, générées par une source superficielle connue. L'espace semi-infini se compose de couches à propriétés élastiques et dynamiques déterminées. Pour résoudre le problème, nous avons choisi une solution numérique mettant en œuvre un programme d'analyse EF d'une suite représentative de couches. Avec les résultats exposés dans l'article, nous proposons un algorithme de prédiction du profil vibratoire d'autres combinaisons de couches.

Keywords: circular footing, harmonic loading, layered system, finite element, dynamic analysis

1 INTRODUCTION

An increasing part of the design of new structures in urban areas is devoted preservation of existing monuments and buildings and maintaining a reasonable environment for the inhabitants near the construction site.

Vibrations, especially during the construction phase, may cause harm to neighbouring structures in terms of increased settlements and cracks and a substantial annoyance to humans in the area. Traffic induced vibrations from new infrastructure developments are also becoming an increasing concern.

An estimate of the impact on the surrounding environment in terms of vibrations is normally required prior to the construction work. However, this is difficult for many reasons. Among others, there is lack of ability to quantify the dynamic properties of the different vibration sources and the transmissibility of the actual soil stratum. In connection with the latter a general numerical model study in comparison to analytical classical solutions has been carried out.

Homogeneous and layered soil conditions are considered. The source is approximated to a circular plate resting on an infinitesimal soil half-space. The plate is subjected to a given vertical harmonic movement. The response of the soil surface at different distances from the plate has been calculated in terms of vibration amplitudes, as an asymmetric problem in a FE formulation. The soil is modelled with different properties. The layers are selected with varying thickness and succession to be representative for normally encountered sand and clay, saturated and unsaturated.

The numerical results are compared with the classical empirical solutions and adjustments are made valid to practicable applications. The paper contains the assumptions, and the results are presented in simple terms where the transmissibility can be estimated with knowledge of the distance to the source and an assumed soil profile.

2 DYNAMIC DISPLACEMENTS OF CIRCULAR FOOTING

A basic parameter to estimate ground vibrations at the soil surface is the dynamic displacement of the circular footing subjected to a vertical cyclic load.

2.1 *Homogeneous soil conditions*

A widely used empirical expression for the homogeneous case, has been proposed by Lysmer and Richart (1966) and it reads:

$$m \cdot \frac{d^2}{dt^2}\delta(t) + \frac{3.4}{1-\mu} \cdot r^2 \cdot \sqrt{G \cdot \rho} \cdot \frac{d}{dt}\delta(t) + \frac{4 \cdot G \cdot r}{1-\mu} \cdot \delta(t) = Q(t) \quad (1)$$

In the Equation (1) m is the mass of the footing with radius r, $\delta(t)$ is the vertical displacement (function of time, t). G and μ are shear modulus and Poisson's ratio, respectively and ρ is the mass density of the perfectly elastic half space. The coefficient to the second term of the left hand side of the differential equation (1) is denoted: $c = 3.4\ r^2(G\rho)^{0.5}/(1\text{-}\mu)$ The coefficient to the last term is the static spring constant: $k = k(G,\mu) = 4Gr/(1\text{-}\mu)$.

In the following only harmonic excitation is considered: $Q(t) = P_o e^{i\omega t}$ where P_o is a real constant (the amplitude of the force) and ω is the circular frequency. The solution in terms of the vertical displacement following Lysmer and Richart, (1966) can be written as $\delta = \delta_o \cos(\omega t+\varphi)$ where φ is the phase shift with respect to the excitation and the displacement amplitude is approximately:

$$\delta_o = (P_o/k)\ ((1\text{-}B\ a_o^2)^2 + (0.85\ a_o)^2)^{-0.5} \quad (2)$$

where $a_o = \omega r(\rho/G)^{0.5}$ (denoted the frequency ratio) and $B = m(1\text{-}\mu)/(4\rho r^3)$. The last factor in Equation (2) is denoted the magnification factor:

$$M(B,a_o) = ((1\text{-}B\ a_o^2)^2 + (0.85\ a_o)^2)^{-0.5} \quad (3)$$

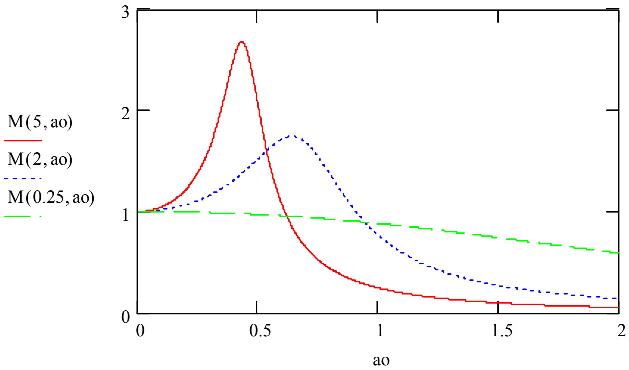

Figure 1. Magnification factor as function of a_o

In Figure 1 the magnification factor is shown for a relevant interval of B: 0.25<B<5.

Equations (1) and (2) are accepted as the solution to the homogeneous case without further discussion. Equation (2) can be written as: $\delta_o/r = p_o\pi rM/k$ where p_o is the excitation expressed as an average stress amplitude on the footing base.

It follows then that the same dimensionless displacement amplitude will occur if r is multiplied by a factor C while the soil properties (G, μ, ρ) and M = M(B,a_o) are unchanged. B is constant if also the mass of the footing is C^3 times greater (e.g. same mass density but all dimensions C times greater). Additionally, a_o is constant if Cω is unchanged.

Shortly, the same dimensionless displacement amplitude is obtained if all length dimensions are multiplied by the same factor (C) but the excitation (in terms of p_o), the soil properties and Cω are kept unchanged.

2.2 *Influence curve, low frequency or static conditions*

Before the case with layered soil is addressed the concept of an influence curve is introduced. Layered soil is treated as a weighted average of a number of solutions where the properties for each layer are assumed valid for the entire half-space.

As an example we take Equation (1) for the static case where $\omega = 0$. All derivatives with t is zero and Q on the right hand side is constant. It is now assumed that G = G(f) and $\mu = \mu(f)$ varies with the dimensionless depth f = d/r where d is the depth of the layer beneath the surface.

For each layer we can calculate a spring constant k(f) according the afore mentioned equation. The value of k =k(f) has the meaning that it is the layer spring constant if the entire half-space had the same values as found in f. With this in mind k can be written as k = k(f) and directly calculated.

There exists a defined value of the spring constant valid for each profile of G and μ that can be used in this static case. This value is denoted $k_s = k_s[G,\mu]$. We use brackets to point out that it depends on the entire profiles of G and μ. It is now assumed that k_s can be calculated as a weighed average of k(f) for each depth as:

$$k_s \approx \left(\int_0^{\infty} i(f) \cdot k(G(f), \mu(f))^{-1}\, df \right)^{-1} \quad (3a)$$

where i(f) is the weight function denoting the influence curve for k^{-1}. In order to obtain correct values in the homogeneous case we have:

$$\int_0^{\infty} i(f)\, df = 1 \quad (3b)$$

Furthermore, we know in beforehand that layers at great depth have little influence on the spring constant as i(f) → 0 for f → ∞. The concept has the obviously fine quality that if one layer with constant properties is increased in thickness it will also have an increasing influence on the result and finally approach the correct solution for the homogeneous case.

A single profile of k(G(f),μ(f)) together with a value of k_s (e.g. calculated by FEM) does not define a profile of i(f) as many shapes of the i-profile may predict the correct settlement.

Schmertmann (1979) has proposed that this curve should be proportional with the stresses beneath the centre line of the footing. A simplified influence curve is proposed based on this concept with a reasonable cut-off in the depth f = 4. This curve has later been improved based on more data (Schmertmann et. al, 1978).

The curve has been further improved by consequently assuming the influence of a certain layer to be proportional with the internal work generated in this layer for the homogeneous case (Denver, 1981).

This enables calculation of both upper and lower bound solutions to the real layered problem. A comprehensive method, which predicts settlements has been presented by Mayne and Poulos (1999) for square footings and embedded footings etc.

Apart from this a rational determination of i(f) is difficult. However, it can be stated that the best choice is the one yielding the best results for a given pattern of usage. This means that this usage should be modelled when i(f) is determined.

2.3 *Layered soil conditions*

As each layer has constant k (denoted k_j for the j'th layer) Equation 3a can be rewritten as:

$$k_s \approx \left[\sum_{j=1}^{n}\left[\left(k_j\right)^{-1}\cdot\int_{f_j}^{f_{j+1}} i(f)\,df\right]\right]^{-1} \tag{4}$$

f_j is the dimensionless depth to the top of the j'th layer. When adopting the above-mentioned cut-off reasonable results are generated when the deepest layer (n) ends at the depth f = 20. From Equation 4 it is seen that the usage is enhanced if the integrated influence curve is available:

$$k_s \approx \left[\sum_{j=1}^{n}\left[\left(k_j\right)^{-1}\cdot\left(I\left(f_{j+1}\right)-I\left(f_j\right)\right)\right]\right]^{-1} \tag{5}$$

In the Equation 5 I(f) is calculated as:

$$I(f) = \int_0^f i(f)\,df \tag{6}$$

If the influence profile is given as an expression I(f) (Equation 6) it is a straight forward matter to calculate k_s by Equation 5. An approximate expression for I(f) is proposed by (Denver, 1981) and given in Equation 7 where the angle is expressed in radians. A cut-off has been adopted as 0≤f≤20.

$$I(f) = \operatorname{atan}(20.5755)-\operatorname{atan}(f+0.5755) \tag{7}$$

3 DYNAMIC DISPLACEMENT OF FREE SOIL SURFACE

The displacement of the free soil surface is normally dominated by the Rayleigh waves (R-waves). They can in fact be regarded as a mixture of compression waves and shear waves where boundary criteria for the free surface are fulfilled. It is well known that R-waves dominates the wave pattern along the surface to a depth of approximately one wavelength for the R-waves.

3.1 *Homogeneous soil conditions*

For homogeneous soil a commonly known expression accounts for both loss of energy due to the spreading (geometrical damping) and absorption in the soil (material damping) is applied (Hall and Richart, 1963):

$$\delta_1/\delta_2 = \sqrt{\frac{a_1}{a_2}}\cdot\exp\left[-\beta\cdot\left(a_2-a_1\right)\right] \tag{8}$$

δ_x is the vertical amplitude at the distance a_x and β is an absorption coefficient.

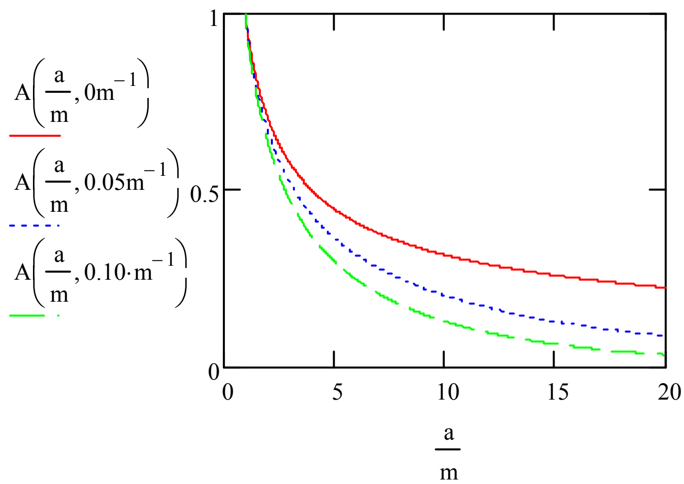

Figure 2. Decay of vertical vibration amplitude

Figure 2 shows $A(a_1,\beta) = \delta_1/\delta_2$ for different values of a_1 for a_2=1 m and for β = 0, 0.05 and 0.10m^{-1} – the latter two values represents end points of a relevant interval of β-values determined experimentally. From Figure 2 it is concluded that the material damping plays a minor role compared with the geometrical damping.

3.2 *Layered soil conditions*

For layered soils we may benefit from the methodology used for Spectral Analysis of Surface Waves (SASW). Here it is applied that the soil layer in the depth of the half wavelength of the shear wave for this soil will nearly completely determine the wave velocity observed on the surface (Heukelom & Foster, 1960).

For a two-layer problem the half wavelengths ($\lambda_1/2$, $\lambda_2/2$) can be calculated (given soil properties and frequency). If $d_{up} = \min(\lambda_1/2, \lambda_2/2)$ and $d_{lo} = \max(\lambda_1/2, \lambda_2/2)$ the resulting surface wave velocity v_R can be expressed as:

$$v_R = w\, v_{R1} + (1-w)\, v_{R2} \tag{9}$$

where v_{R1} and v_{R2} are the Reyleigh wave velocities in the upper and lower layer, respectively. The value of w = [0|1] is shown in Figure 3.

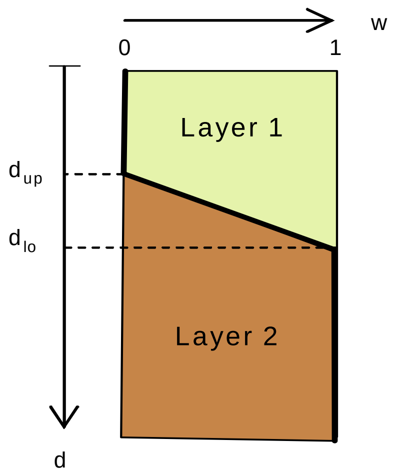

Figure 3. Influence (w) of different layer properties on surface wave velocity as a function of depth to interface (d)

4 COMPARISONS OF ANALYTICAL AND FE RESULTS

In order to verify the proposed model a number of FE calculations are carried out investigating the dynamics of the two-layered soil system for a circular footing subjected to dynamic force.

The analyses are performed in time domain using Plaxis Dynamic Module employing absorbing boundary formulation at the side and the bottom of the model simulating radiation damping at infinity. In the analyses the soil is assumed to be linear, homogeneous, elastic medium. Material damping is included in the form of Rayleigh damping.

An axisymmetric model is constructed with a rigid circular footing with radius r=1.0 m. 15 node FE elements are used and the mesh is designed as a function of the soil properties and the considered predominant frequency of the dynamic excitation load as given in table 1.

As the absorbing boundaries applied in Plaxis are best suited to high frequency vibrations but not so good for absorbing the energy from the low frequency vibrations investigated by Kellezi (2000), the size of the FE model is chosen such that the possible reflections from the far field are minimized.

The two layers system consists of a weaker layer (Soil I) overlain a stronger layer (Soil II) as shown in Figure 4. The FE-net is shown in Figure 5.The soil properties of the two layers are shown in table 1.

Table 1: Soil properties

Footing (concrete)		**Excitation**	
Radius (r_o)	1 m	$p = p_o \sin(\omega t)$	
Height	0.5 m	p_o = 10 kPa	
Unit weight	24 kN/m3	f = 10 Hz	
	Soils		
		Soil I	Soil II
Young's modulus (E)		10 MPa	30 MPa
Poisson's ratio (μ)		0.3	0.5
Unit weight (γ)		15 kN/m³	20 kN/m³
Damping (α, β)		0.001, 0.01	0.001, 0.01

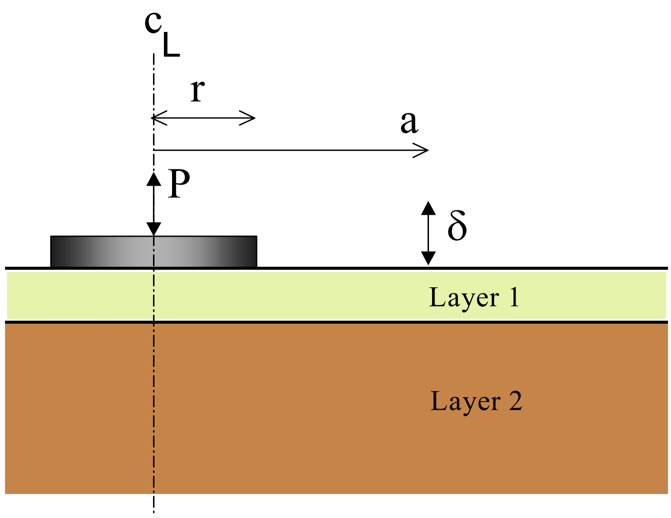

Figure 4. Calculated example with two layers

The dynamic analysis is considered as a single source vibration problem. Points at the footing centre and at different distances from the footing are chosen where the dynamic vertical displacement as a function of time is investigated.

From the start the model is built up with different layering with thicknesses d_i increasing from 0 m, to 1m, 2m, 4 m, 6m, 8 m, 10 m. The first and the latter apply to homogeneous half-space case.

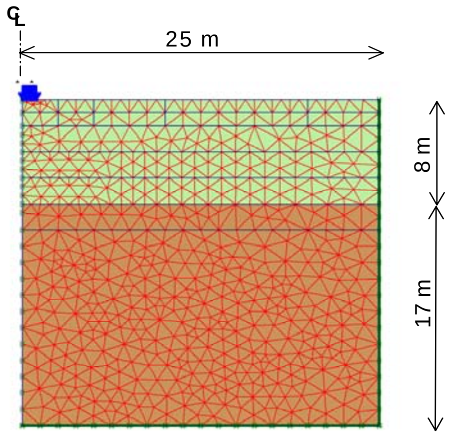

Figure 5, FE dynamic model. Same element net used for all values of di. In the figure Soil I shown green for d_i = 8 m

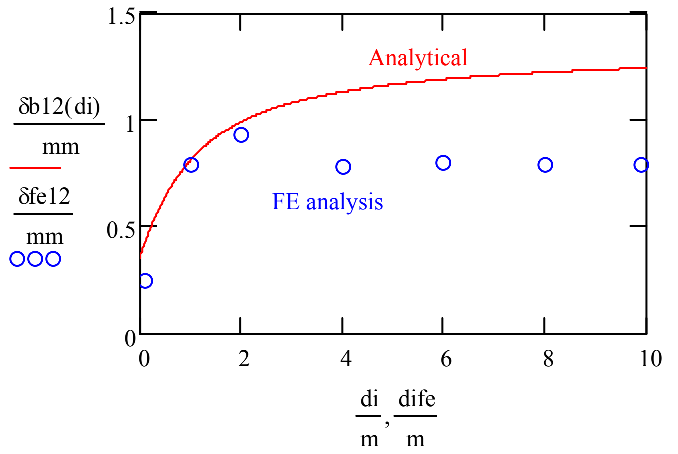

Figure 7. Calculated footing vertical vibration amplitude for the two-layer soil system.

4.1 *Displacement of footing*

The results from the FE analyses are shown in Figure 6 as vertical footing vibration for time duration of 0.5 s and varying layer thicknesses. Layer thicknesses d_i = 10 m, 6 m, 2 m, 1 m, 0 m are chosen for the plot. 3% material damping is included in the soil.

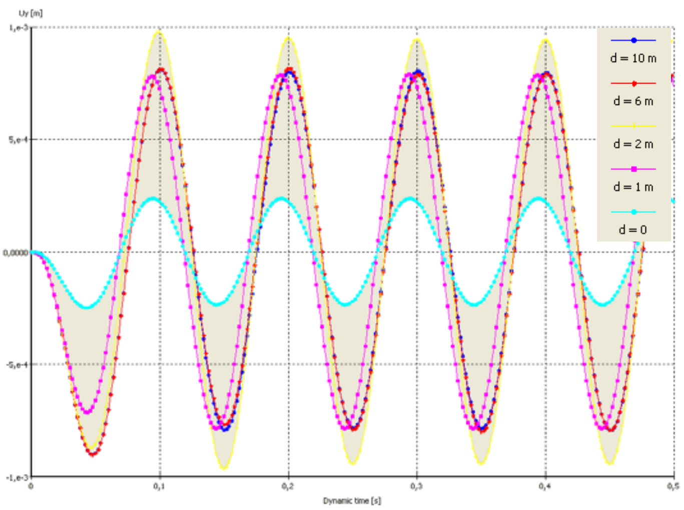

Figure 6, Vertical footing vibration for varying layer depth.

Amplification of footing vibration is observed for d = 2 m, which is expected to correspond to resonance frequency of the layer, see Kellezi and Nielsen (2000).

Furthermore, the vertical displacement amplitude has been estimated by the analytical procedure given in section 2. k is calculated by Equations 5–7 and a_o and B by Figure 3 and Equation 9.

In Figure 7, δ_{b12} means vibration amplitude (displacement) for the combination Soil I over Soil II. The curve represents prediction by the analytical procedure whereas points correspond to FE calculations (with material damping).

There is a difference in the analytical and the FE results for the large depth of the Soil I. There is also a change in the shape of the curves given in Figure 7.

4.2 *Displacements at the free soil surface*

The FE vertical displacement amplitudes have been calculated for a number of points with different distances a to the centre of the footing and different values of d_i. The results are shown relatively to the displacement of the footing for the same value of d_i (A_{FE}=$\delta_{FE\ free\ surface}$ / δ_{FE12}) in Figure 8 without and with material damping.

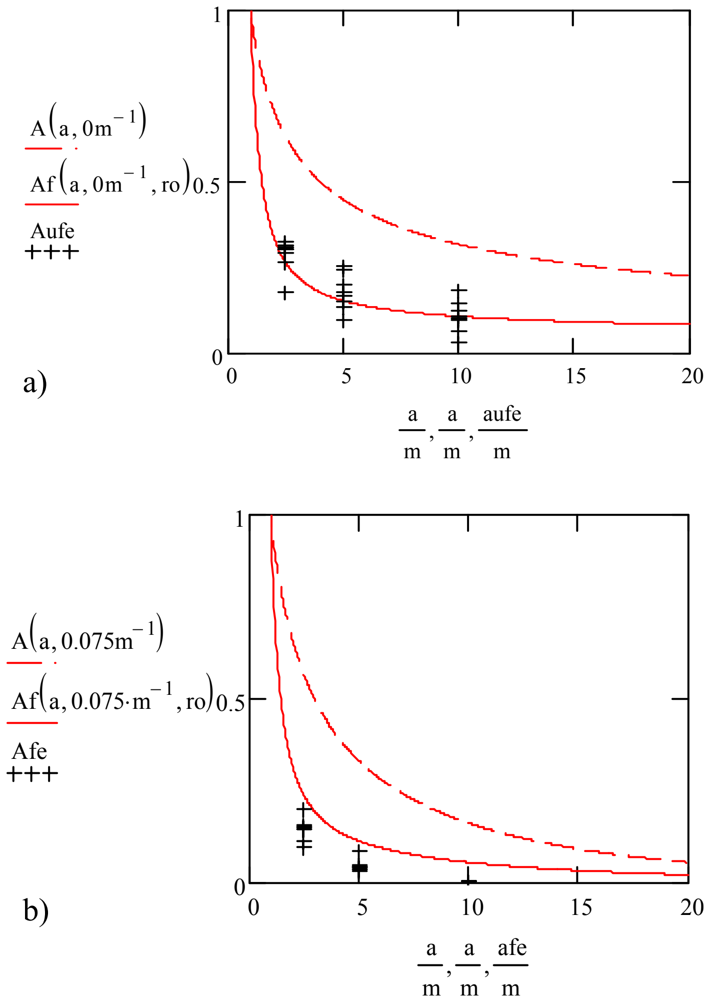

Figure 8. Calculated vertical vibration amplitude divided with the amplitude of the footing for different a
a) no material damping b) with material damping

The different points for the same distance a, represent different values of d_i calculated by FE. The upper curves (punctuated) in Figure 8 give amplitudes estimated by Equation 8. Equation 10 given in the discussion section represents the lower curves

5 DISCUSSION

5.1 *Displacement of footing*

The FE amplitudes of the footing vertical vibrations given in Figure 7 are smaller than calculated by the proposed analytical procedure. The reason is twofold:

The algorithm proposed by Lysmer and Richart (1966) for homogeneous soil (d_i = 0 and 10 m) does not include material damping. As material damping is included in the FE-calculations smaller displacements are obtained.

At depth of about d_i = 2 m relatively greater values are observed as at these depths compression waves (p-waves) reflected by the interface between the layers are approximately in phase with the excitation. This produces a larger amplification than predicted by the analytical model (as this effect is not included). However, the prediction is still on the safe side in the sense that greater displacements are predicted.

5.2 *Displacements at free soil surface*

The displacements of the free surface (relatively to the displacement of the footing) are significantly smaller than calculated by Equation 8. The reason may be that the decay in amplitude for greater distances is estimated as the energy loss of the dispersion of pure surface waves (Rayleigh waves). This corresponds to the power (0.5) of the first factor of Equation 8. As body waves correspond to a greater power a modified equation is proposed as given in Equation 10.

$$Af(a,\beta,ro) := \left(\frac{ro}{a}\right)^{1.5\cdot\left(\frac{ro}{a}\right)^{0.5}+0.5}\cdot\exp[-\beta\cdot(a-ro)] \quad (10)$$

The power in this equation starts with 2 near the footing and approaches 0.5 at greater distances. The second factor representing the material damping is unchanged.

The estimated relative displacements are calculated using Equation 8 and shown in Figure 8. The result are in excellent fit for the case without damping, (Figure 8a) indicating that the geometric damping is correctly modelled. In the case with damping, (Figure 8b) too great amplitudes are still predicted. The reason may very well be that the used material damping in the Equation 8 (β=0.075m^{-1}) does not correspond exactly with the values applied in the FE analyses. In order not to under predict the displacement Equation 10 is recommended on the basis of the results of this study.

6 CONCLUSIONS

From the above investigation the following conclusions can be drawn:

A simple analytical interpolation procedure predicting the footing vertical vibration amplitude for layered soil has been proposed. A series of FE model calculations confirmed the procedure in terms of reasonable results. However more research is needed further developing the procedure taking into account material damping and the resonance effect for particular layer depths.

The decay of vertical vibrations amplitudes at the free surface for greater distances is substantially greater than predicted by the traditional Equation 8. A modified analytical expression, Equation 10, has been proposed on basis of the present study.

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