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1*g* model tests with foundations in sand

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ABSTRACT: This paper presents the results of a series 1*g* model tests with both a circular and a strip foundation on dense sand. The test results have been compared with the results from finite element calculations based on a non linear Mohr-Coulomb yield criterion taking into account the dependence of the magnitude of the friction angle on the stress level at the instant of failure. Good agreement between test results and numerical calculations has been found.

1 INTRODUCTION

The main problem of 1*g* model test in soil is the fact, that the results cannot immediately be extrapolated to a full scale structure, because of the non linear behaviour of the soil leading to scale effects, which are difficult to deal with. Some of the scale effects are due to the fact, that it is impossible to obtain stresses of the same magnitude in corresponding points in the model and in the prototype. Today the only way that this can be achieved is by using a geotechnical centrifuge, of which the purpose is to increase the gravitational field in order to make up for the otherwise smaller stresses in the model. The desire to obtain the same stress level in the model and the prototype stems from the fact, that the strength of the soil is highly dependent upon this so an immediate linear extrapolation of results from the model to the prototype becomes impossible without this condition fulfilled.

Over the past ten to twenty years there has been a tremendous increase in computational power due to bigger and cheaper computers, and this development has made it possible to base the design of geotechnical structures on more sophisticated and realistic soil models, than the linear Mohr-Coulomb model, which by far has been the most used soil model. In general, centrifuge testing can by no means be replaced by 1*g* testing, but in cases where it is a matter of verifying a geotechnical design based on a new soil model 1*g* model testing may be applicable provided the dependence of the soil parameters with the stress level is included in the model.

The main purpose of the present project is to show, that by using a slightly more advanced failure criterion than the linear Mohr-Coulomb model, one can obtain results from theoretical analysis, which are in very good accordance with simple gravity model tests.

Table 1. Properties of Esbjerg Sand.

Parameter	value
D_{10} (mm)	0.25
D_{60} (mm)	0.58
$C_u = D_{60}/D_{10}$	2.32
D_{50} (mm)	0.50
Specific density	2.621
Maximum void ratio $e_{\max}$	0.733
Minimum void ratio $e_{\min}$	0.449
Relative density in tests, D_r	0.84
Dry unit weight in tests (kN/m^3)	17.54

2 SAND USED IN TESTS

The sand used in the tests is Esbjerg sand, which is an alluvial, medium grained sized quartz sand of subangular shape with the characteristics given in Table 1.

To find the strength properties of the sand triaxial tests were carried out and these tests are described in detail elsewhere, (Krabbenhoft, unpubl.); and the main results are given in Figure 1 and Figure 2 and Table 2.

It is a well known fact, that dilation plays an important role in the study of the strength of soil (Manzari and Nour, 2000) and also in general, the angle of dilation is considerably smaller than the angle of friction. Collapse loads for materials with a nonassociated flow rule are smaller than those obtained for the same material when an associated flow rule is assumed. From experience it is known, that in cases where there is a substantial difference between φ and ψ numerical problems arise in the solution of nonlinear finite element equations. To overcome these difficulties Drescher & Detouney (1993) proposed using a

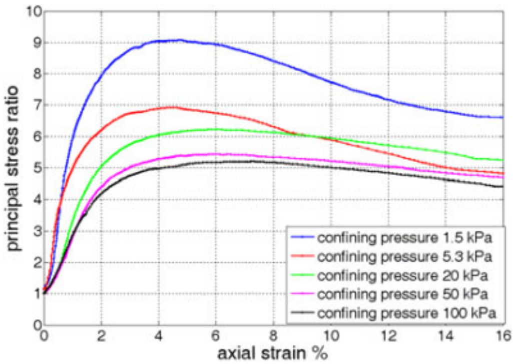

Figure 1. Principal stress ratio, σ_1/σ_3.

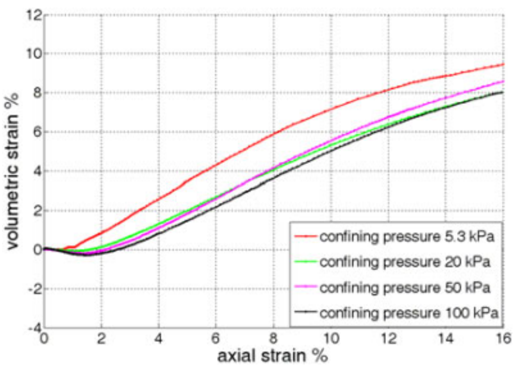

Figure 2. Volumetric strains, ε_v.

Table 2. Test results for Esbjerg sand, $D_r = 0.84$.

Confining pressure, kPa	Peak angle of friction φ_{peak}, degrees	Angle of dilation ψ_{max}, degrees	Modified peak angle of friction φ_{mod}
1.5	53.3	23.0*	47.0
5.3	48.6	18.3	43.0
20	46.1	14.8	40.1
50	42.4	15.5	38.7
100	41.3	13.7	37.6

*estimated value

modified friction angle making it possible to deal with the material, as if it were obeying the normality condition. According to Davis (1968) the modified friction angle φ_{mod} can be found from the equation:

$$\tan\varphi_{mod} = \frac{\sin\varphi\cos\psi}{1-\sin\varphi\sin\psi} \tag{1}$$

The values of φ and ψ on the right hand side of the equation are given in column 2 and 3 in Table 2 and the values of φ_{mod} – used in the numerical calculations – are given in column 4 in Table 2.

Table 3. Parameters in nonlinear yield function.

State of stress	k_0	s_{c0}, kPa	a
Axissymmetry	3.9355	19.7073	1.4330
Plane strain	4.876	29.0391	2.330

The values of the friction angle in Table 2 – fourth column – are used when computing the bearing capacity of the circular foundation as the state of stress in the soil for this case is normally assumed to be equivalent to triaxial conditions i.e. the principal stress ratio b defined as:

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \tag{2}$$

is equal to zero as $\sigma_2 = \sigma_3$. For the plane strain condition $\sigma_2 > \sigma_3$ and in this case there is an increase in φ. According to Bonding (1977) the value of φ in plane strain can be estimated from the equation

$$\varphi = (1 + 0.16D_r)\varphi_{tr} \tag{3}$$

where D_r is the relative density and φ_{tr} is the triaxial friction angle. Equation (3) is consistent with the findings of Wang & Lade (2001) for dense sand. Accordingly – for the computation of the bearing capacity of the strip foundation – the friction angles have been found by multiplying the values of the fourth column in Table 2 by the factor 1.13, which has been found by inserting D_r from table 1 into Eq. (3).

3 THE NON LINEAR FAILURE CRITERION

Several authors have suggested yield functions that take the φ – dependency of stress level into account, e.g. De Mello (1977), Charles & Watts (1980), Simonini (1993), Baker (2004). In the present study it has been found appropriate to use an expression of the form:

$$\sigma_1 = k_0\sigma_3 + s_{c0}\left(1 - \exp\left(-a\frac{\sigma_3}{s_{c0}}\right)\right) \tag{4}$$

which is a curved envelope, that passes through the origin and tends toward the asymptote:

$$\sigma_1 = k_0\sigma_3 + s_{c0} \quad \text{for} \quad \sigma_3 \to \infty \tag{5}$$

The parameters k_0 and s_{c0} defines the asymptote slope and intersection with the σ_1 axis, respectively, and a adjusts the curvature. The parameters k_0, s_{c0}, and a are determined from nonlinear regression analysis based on the modified friction angles described above. The values are given in Table 3.

4 FINITE ELEMENT CALCULATIONS

The yield criterion of Eq. (3) is implemented into a finite element code. The material is considered as a linearly elastic – perfectly plastic material. For the plastic stress update a method analogous to the one for a Hoek-Brown material is employed, see (Clausen & Damkilde 2008). The footing is assumed to be rigid and perfectly rough. A downwards displacement is applied to the footing nodes in steps until the failure load is reached. The footing load is then calculated by summing up the vertical reaction forces of the footing nodes and dividing with the footing area.

5 THE MODEL TESTS

To verify the validity of the proposed nonlinear yield function, 1*g* model tests in axisymmetric and plane strain conditions were carried out. The sand used for the tests was the Esbjerg sand described above, and the tests were conducted at a relative density $D_r = 0.84$. A circular footing with diameter $B = 10$ cm was used for the axissymmetric case and for the plane strain case the dimensions of the foundation were: $B \times H \times L = 10\,\text{cm} \times 10\,\text{cm} \times 40\,\text{cm}$. For both foundations the base was covered with a rough material, to make it perfectly rough. In all tests the footing was resting on the surface, resulting in no overburden pressure. The dry sand was placed in a container in layers of approximately 5 cm. Each layer was tamped a certain number of times to give the desired relative density and the total volume of sand was weighed. To minimize the effect of the sand not having a completely uniform density throughout the container, ten identical tests of each foundation were performed. For the circular foundation the container was a cylindrical shape with diameter = 55 cm and height = 35 cm and for the plane strain tests a rectangular wooden box with a length = 80 cm, width = 40 cm and depth = 24 cm was used. The inner surface of the box was covered with a plastic foil to minimize friction between the ends of the foundation and the sides of the box. Friction tests carried out separately showed that the coefficient of friction μ could be taken = 0.32.

The load was applied to the footing by a hydraulic jack mounted on a steel beam, which was fastened to the concrete floor in the lab using 28 mm threaded steel bars. During the test the load was recorded using a load transducer, HBM S9, and the vertical displacements were recorded by a displacement transducer HBMWA/50 mm. Both load- and displacement transducers were calibrated before the tests. The load was raised continuously, and the rate of displacement was app. 5 mm/minute. The test values were recorded by means of a datalogger Spider 8. Bulging was observed but the bulge never reached the edge of the container, indicating that the size was adequately large, in order to have only minor influence on the test results.

6 ANALYTICAL CALCULATION OF THE BEARING CAPACITY OF A SHALLOW FOOTING

The ultimate bearing capacity of a rough, shallow, circular foundation resting on a cohesionless material is traditionally found from the following equation – given in several textbooks:

$$p_f = 0.5\gamma B N_\gamma s_\gamma + q N_q s_q \tag{6}$$

where p_f = bearing capacity pressure, γ = effective unit weight of soil, B = diameter of foundation, N_γ and N_q = bearing capacity factors which are functions of the soil friction angle φ, s_γ and s_q = shape factors which are 0.6 and 1.2 for a circular foundation and 1.0 and 1.0 for a strip foundation, q = effective overburden pressure.

In the tests in this project, the footings are initially resting on the surface of the sand; i.e. the contribution from the overburden pressure to the ultimate load is zero at the beginning of the test, but as the load is increased, the footing sinks into the ground introducing a vertical effective stress at the foundation level, which cannot be ignored, as it accounts for a significant part of the bearing capacity of the foundation.

The bearing capacity factors N_q and N_γ can be found from the below equations – the expression for N_q presented by Reissner (1924) and for N_γ by Caquot and Kerisel (1953):

$$\begin{aligned} N_q &= \exp(\pi \tan\varphi)\tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right), \\ N_\gamma &= 2(N_q + 1)\tan\varphi \end{aligned} \tag{7}$$

The overburden pressure at the foundation level can be found as $q = \delta\gamma$ where δ is the vertical displacement of the foundation and γ the density of the sand. Inserting (7) in (6) yields the following equation for the bearing capacity:

$$\begin{aligned} p_f &= \gamma B \tan\varphi s_\gamma + \\ &\exp(\pi\tan\varphi)\tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)(\gamma B \tan\varphi s_\gamma + \delta\gamma s_q) \end{aligned} \tag{8}$$

With known values of p_f and the vertical displacement δ, the value of the mobilized friction angle φ can be found from (8); the current values of N_q and N_γ can be found from (7) and finally the contribution from the overburden pressure can be deducted from p_f in (6) to produce the bearing capacity due to the selfweight of the soil.

7 DEFINITION OF FAILURE LOAD

For shallow foundations, three failure modes have been described by Vesic (1973), amongst others. The relevant failure mode in the present study is general

Table 4. Results from tests and FEM analysis.

Footing	Circular	Strip
Failure load from tests [kPa]	148	255
Failure load from FEM [kPa]	151	277

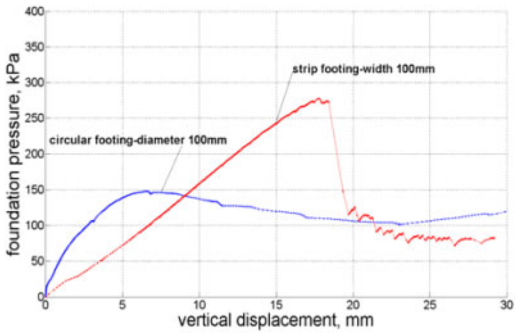

Figure 3. Load-displacement graphs.

shear failure, which is expected to take place for a relative density $D_r = 0.84$. When the footing fails in general shear, there is no doubt as to the magnitude of the failure load, as the load-displacement curve displays a pronounced peak. Table 4 shows the test values together with the results from the finite element calculations. The test values are taken as the average of ten tests for each type of foundation. Typical load-displacement graphs are shown in figure 3.

8 CONCLUSIONS

Triaxial tests carried out on dense Esbjerg Sand at low stress levels show, that the triaxial angle at peak depends strongly on the stress level. Because the friction angle is dependent on the confining pressure, the linear Mohr – Coulomb yield criterion is ill-suited for the determination of the failure load of 1g model scale footings. Therefore – on the basis of the results from the triaxial tests – a non linear Mohr-Coulomb yield criterion has been proposed and implemented in a finite element program. To overcome numerical difficulties due to the non-associative behaviour of sand the associative flow rule is used, but with yield parameters modified with the equation given by Davis. Results from the finite element analysis of the bearing capacity of both 100 mm circular and strip footings show a good agreement with results obtained from simple model-scale footing tests.

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