Theoretical modelling of cycling using HARM

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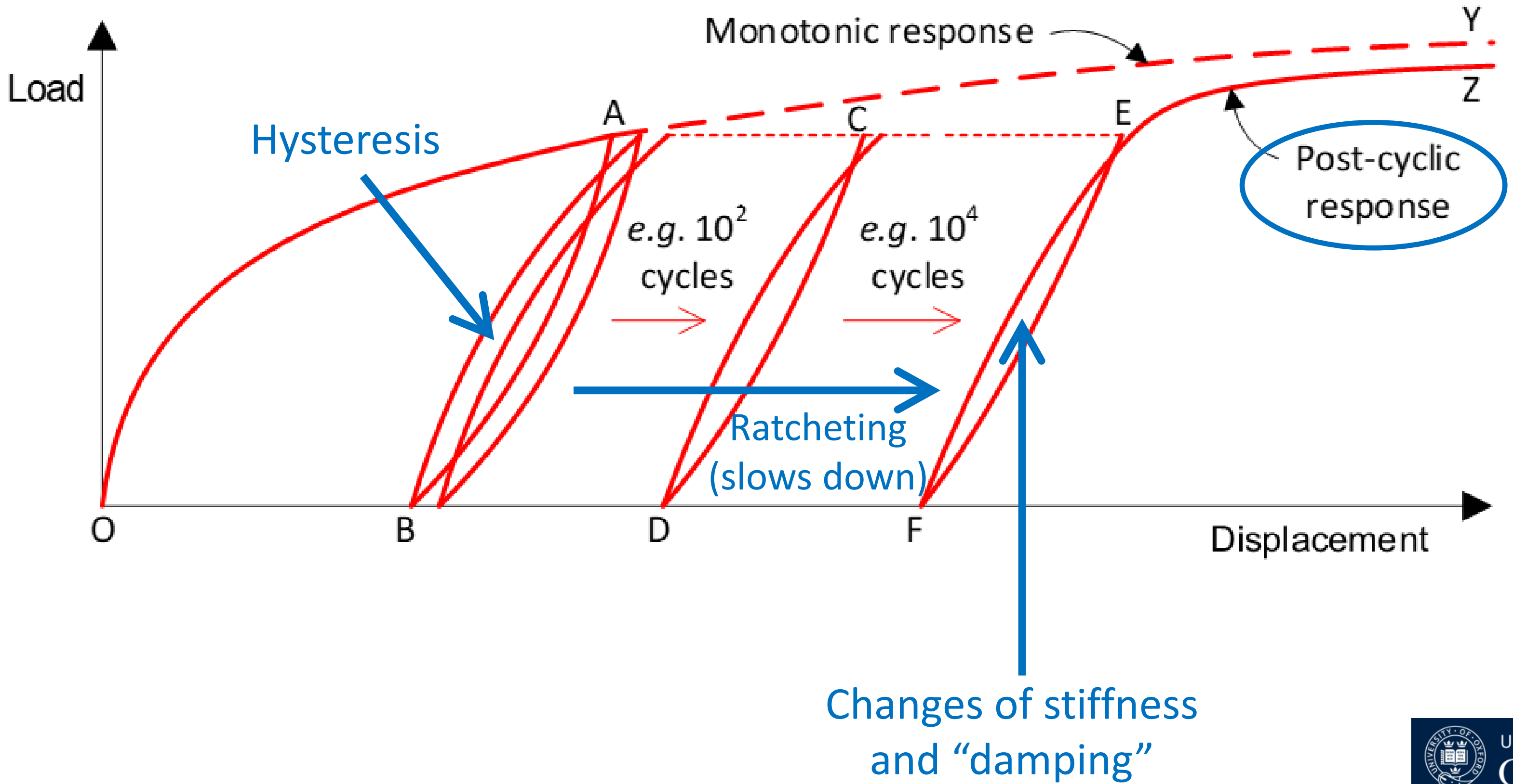
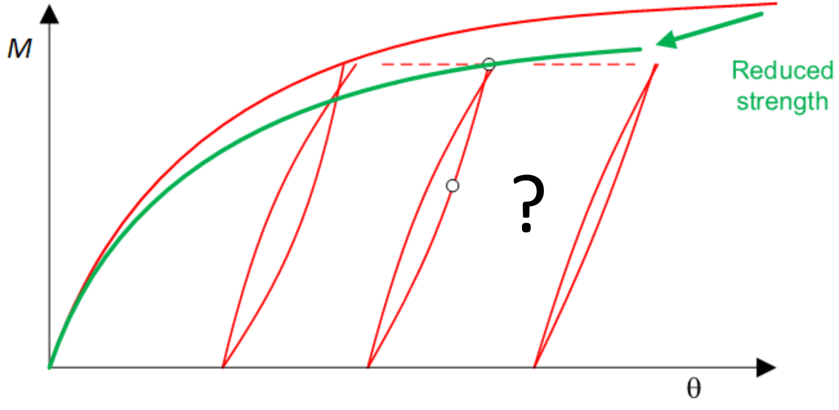
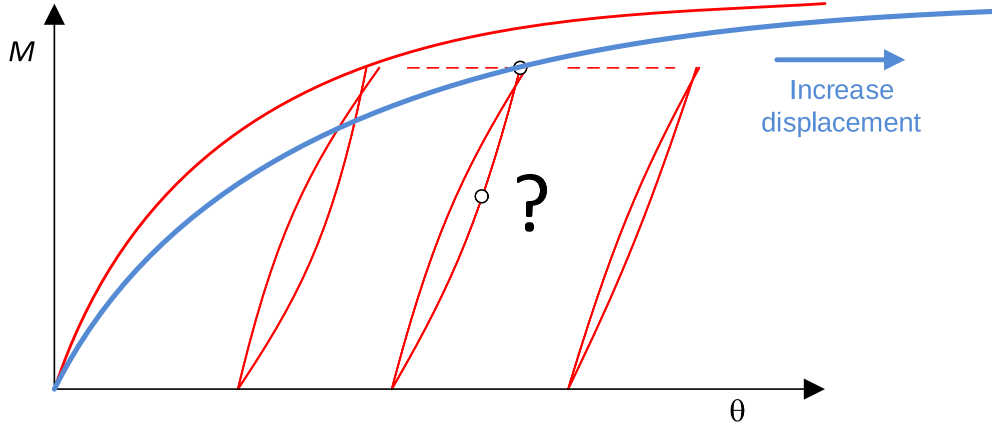
Phenomena to be modelled during cycling



Typical “scaling” design methods for cycling (unsatisfactory)

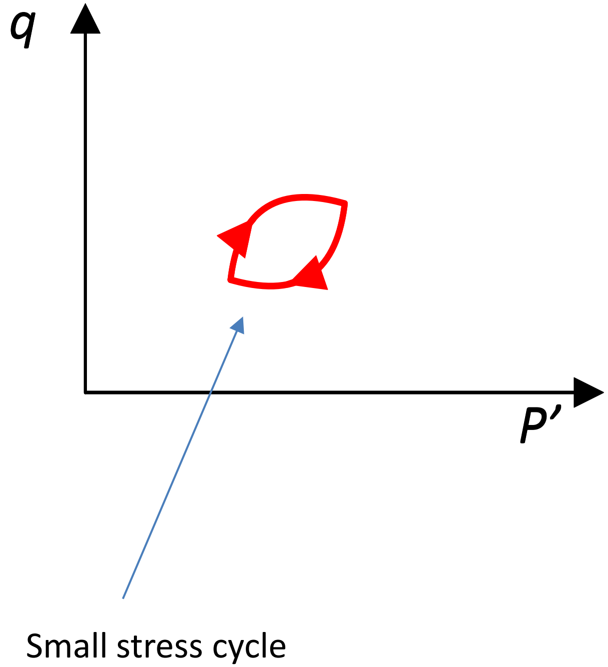


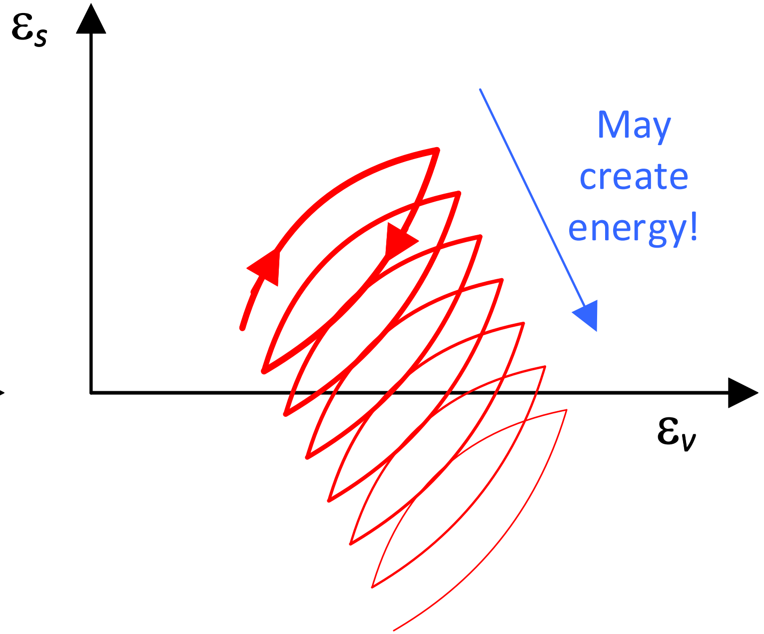


Reduced strength at constant stiffness

Increased displacement at constant strength

Why is thermodynamics important in cycling?





Hyperplasticity

Thermodynamics
First and Second Laws

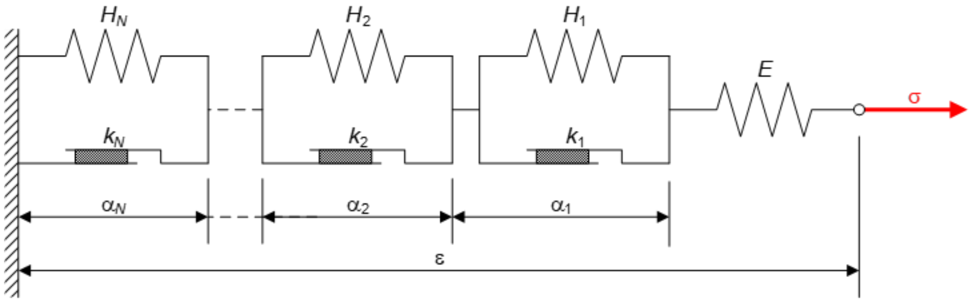
Two scalar functions:
Stored energy = f(strain, plastic strain)
Dissipation = d(plastic strain rate)

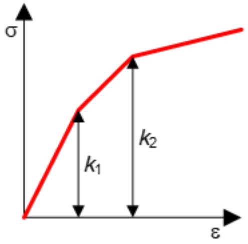
Some standard maths

Plasticity model:
Stored energy = f(strain, plastic strain)
Yield surface = y(stress, etc)

Model non-linearity in cyclic loading

Modelling cycling with "multisurface plasticity"

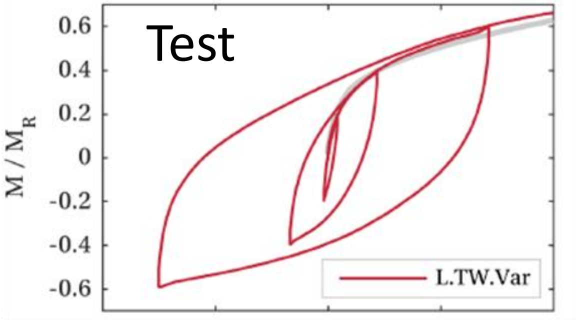


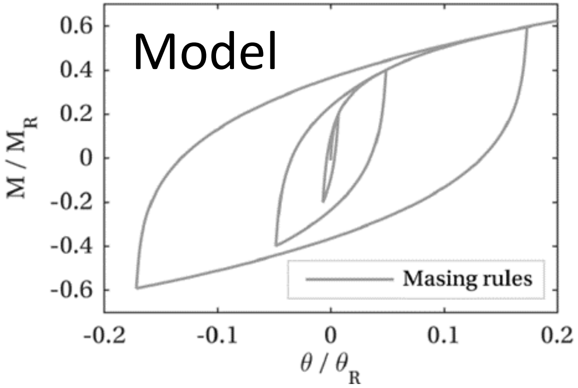


$$f = \frac{E}{2}\left(\varepsilon - \sum_{n=1}^{N} \alpha_n\right) + \sum_{n=1}^{N} \frac{H_n}{2}\alpha_n^2$$

$$d = \sum_{n=1}^{N} k_n \left|\dot{\alpha}_n\right|$$

.... automatically obeys "Masing rules" on cycling

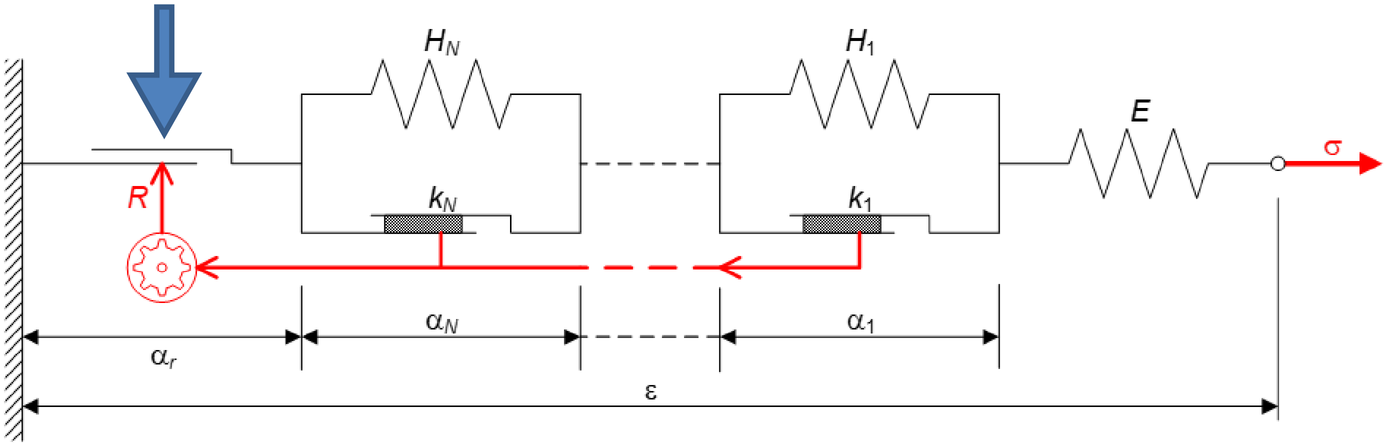


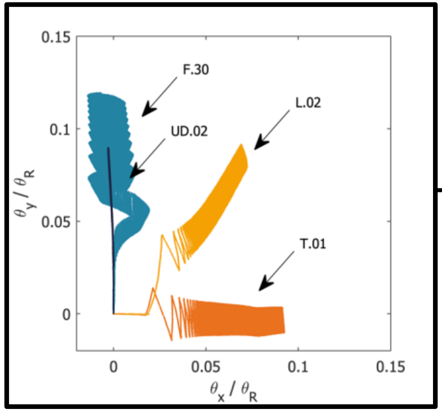


Figures from Richards (2019)

Modelling ratcheting: the "HARM" approach

Add a special "ratcheting element", giving a small additional displacement





$$d\alpha_r = \text{sign}(\sigma) \sum_{n=1}^{N} R_n \left| d\alpha_n \right|$$

Magnitude of plastic strain

Ratcheting strain

Direction of the stress

Small fraction *e.g.* 10^{-4}

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Scaling of ratcheting (“Acceleration”)

$$d\alpha_r = \boxed{R_{fac}}\,\mathrm{sign}(\sigma)\sum_{n=1}^{N} R_n \left|d\alpha_n\right|$$

Complete model definition

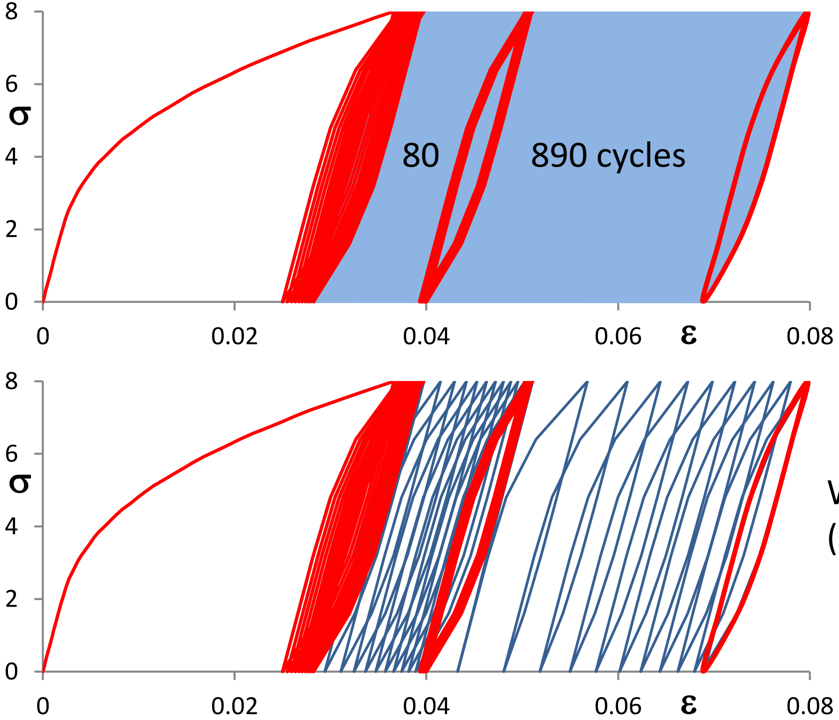
$$f = \frac{E}{2}\left(\varepsilon - \sum_{n=1}^{N}\alpha_n - \alpha_r\right)^2 + \sum_{n=1}^{N}\frac{H_n}{2}\alpha_n^2$$

$$d = \sum_{n=1}^{N} k_n\left|\dot{\alpha}_n\right| + \left(\sigma\dot{\alpha}_r\right)$$

$$+\Lambda_r\left(\dot{\alpha}_r - \boxed{R_{fac}}\,S(\sigma)\sum_{n=1}^{N} R_n\left|\dot{\alpha}_n\right|\right)$$

$$+\Lambda_h\left(\dot{\beta} - \left|\dot{\alpha}_r\right|\right)$$

$$R_n = f(\beta)$$



No acceleration (1000 cycles)

With acceleration (50 cycles)

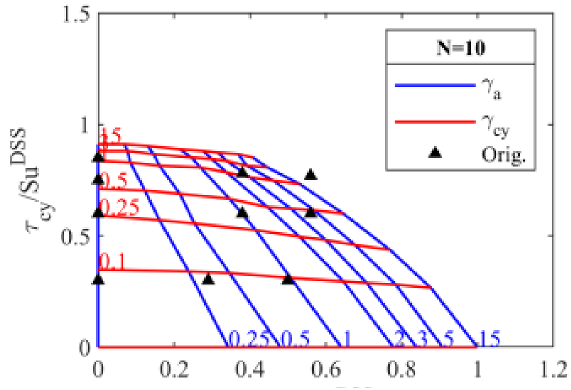
Continuum models (1): Clay

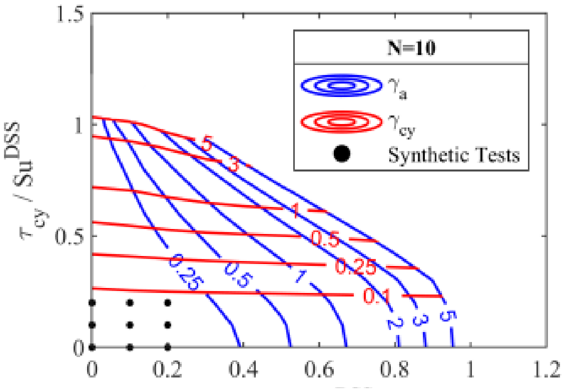
– Develop a model for undrained clay in Direct Simple Shear
  - Includes rate effects and ratcheting

– Use model to compare with NGI "contour diagram" approach for predicting strains during cycling
  - Create artificial contour diagrams from the model to compare with NGI

Compare with NGI contour diagrams

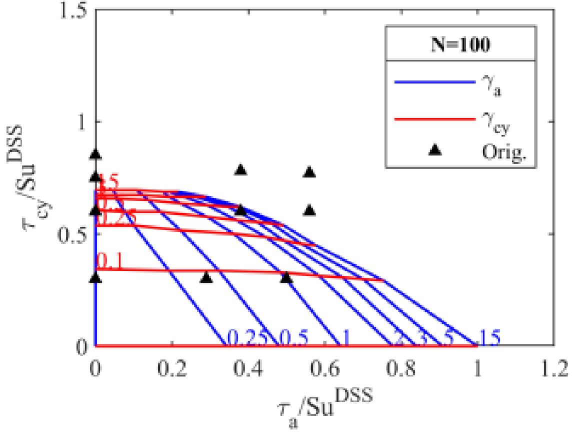
N_{cycles} = 10

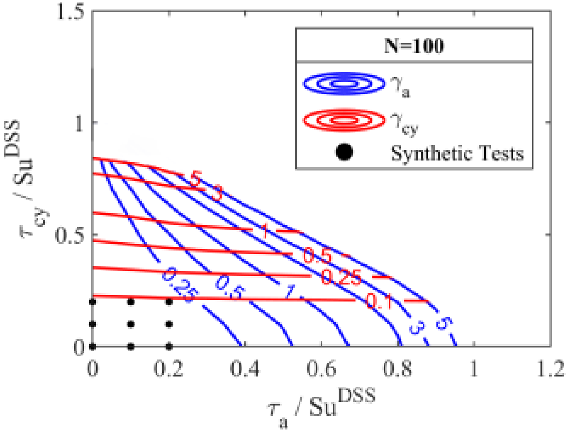




Modelling by Toby Balaam

N_{cycles} = 100





NGI tests on Drammen Clay: empirical contour diagram

Synthetic contour diagram generated by HARM model

Continuum models (2): Sand

- Develop “HySand” model for sand in triaxial tests
- Compare with Torsten Wichtmann’s database of cyclic tests

- Generalised to general stress states
- Impemented in FE codes ABAQUS and PLAXIS

HySand – a constitutive model for sand under cyclic loading

Hyperplasticity: entire model defined by two functions:

$$g = -\frac{p_r}{k_r(1-m)(2-m)}\left(\frac{p_0}{p_r}\right)^{2-m} - \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{\sigma}:\boldsymbol{\alpha}^{(n)} - \frac{1}{N}\sum_{n=1}^{N}\frac{I_{1\sigma}}{3}\alpha_{pc}^{(n)}$$

$$y^{(n)} = \frac{4I_{1\sigma}J_{2\mathrm{X}^{(\mathrm{n})}} - 3\mathrm{tr}\left(\boldsymbol{\sigma}\mathbf{X}^{(\mathrm{n})^2}\right)}{8\left(\frac{n}{N}\mu\right)^2 I_{3\sigma}} + \left(\frac{N\chi_{pc}^{(n)}}{p_c^{(n)}}\right)^r - 1 = 0$$

$$\mathbf{X}^{(\mathrm{n})} = \cdots, \qquad p_0 = \cdots, \qquad \beta = \cdots, \qquad \text{etc.}$$

k_r	g_r	m	φ_c	β_{max}	Β	Γ	Δ	λ_Γ	λ_Δ	$A_{\max}$	h_0	b	r
516	400	0.7	33.1	1.0	2.057	1.98	1.677	0.0032	0.0008	40	750	3	0.3

Work by Luc Simonin

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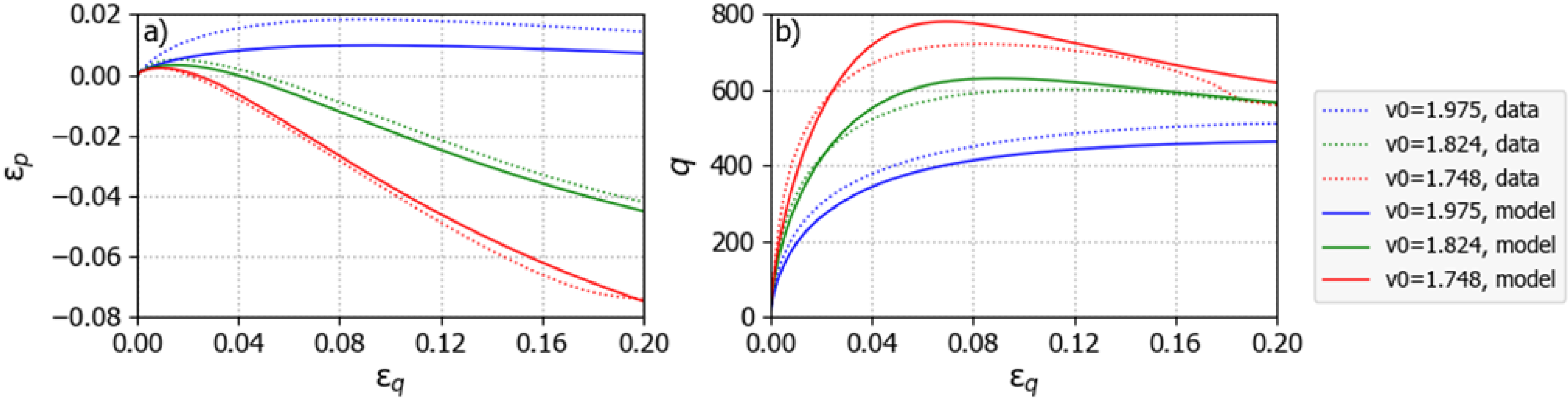
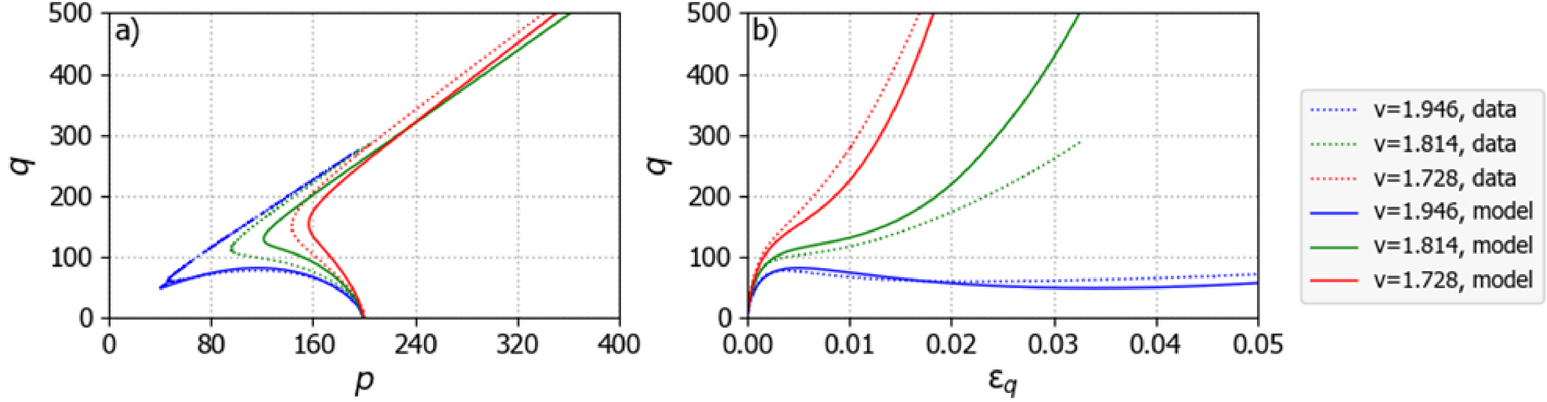
Monotonic tests

Drained tests at different densities



Undrained tests at different densities



Data by Wichmann,modelling by Simonin

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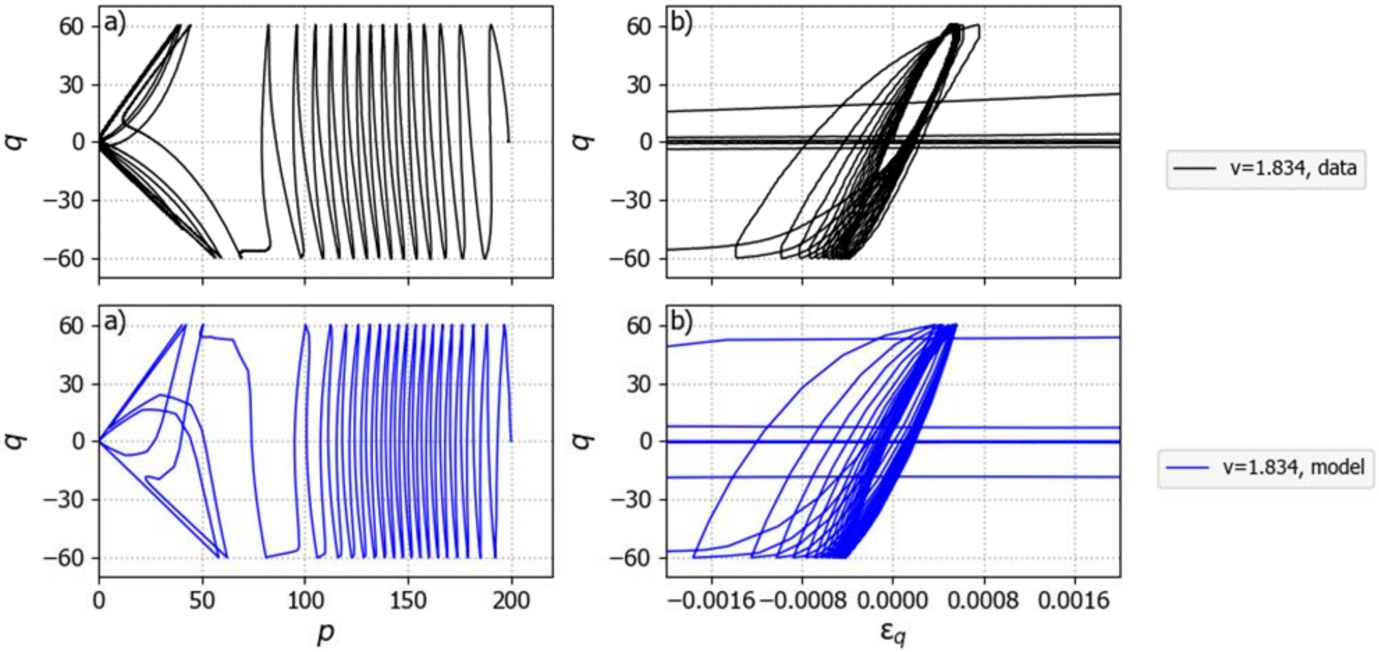
Cyclic tests: stress controlled

Data

Model



Data by Wichmann,modelling by Simonin

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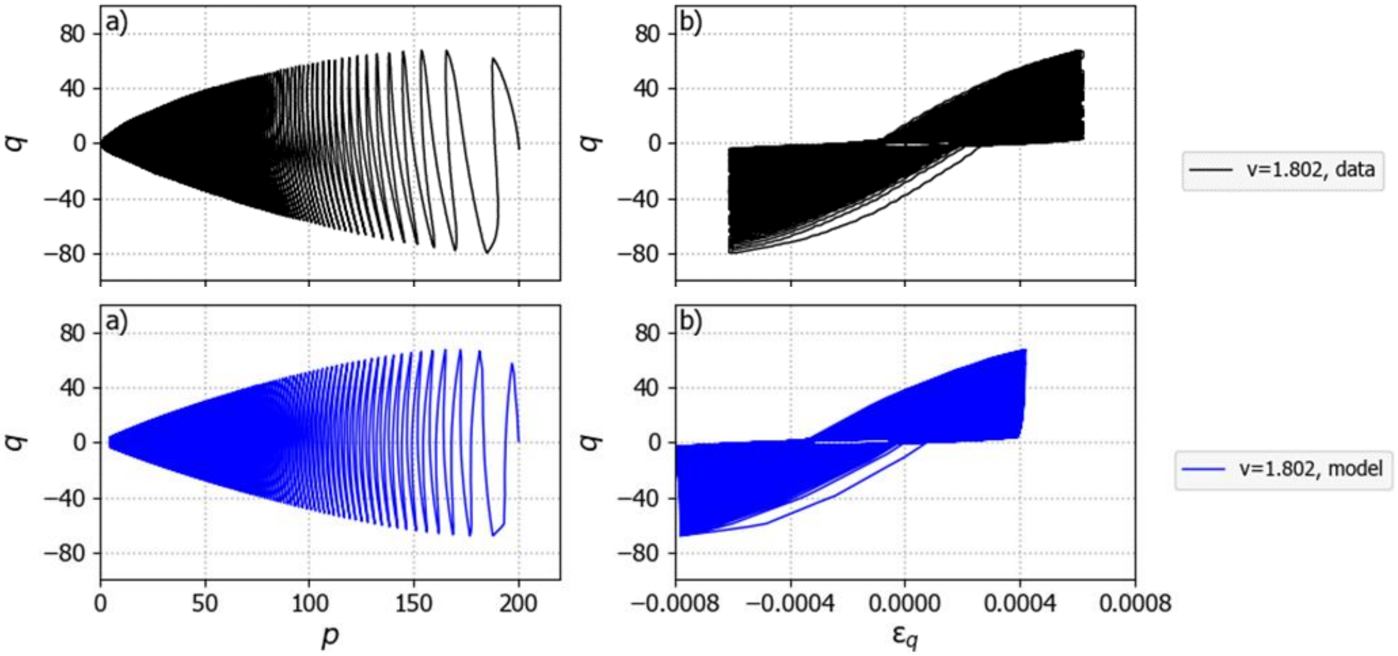
Cyclic tests: strain controlled (small cycles)

Data

Model



Data by Wichmann,modelling by Simonin

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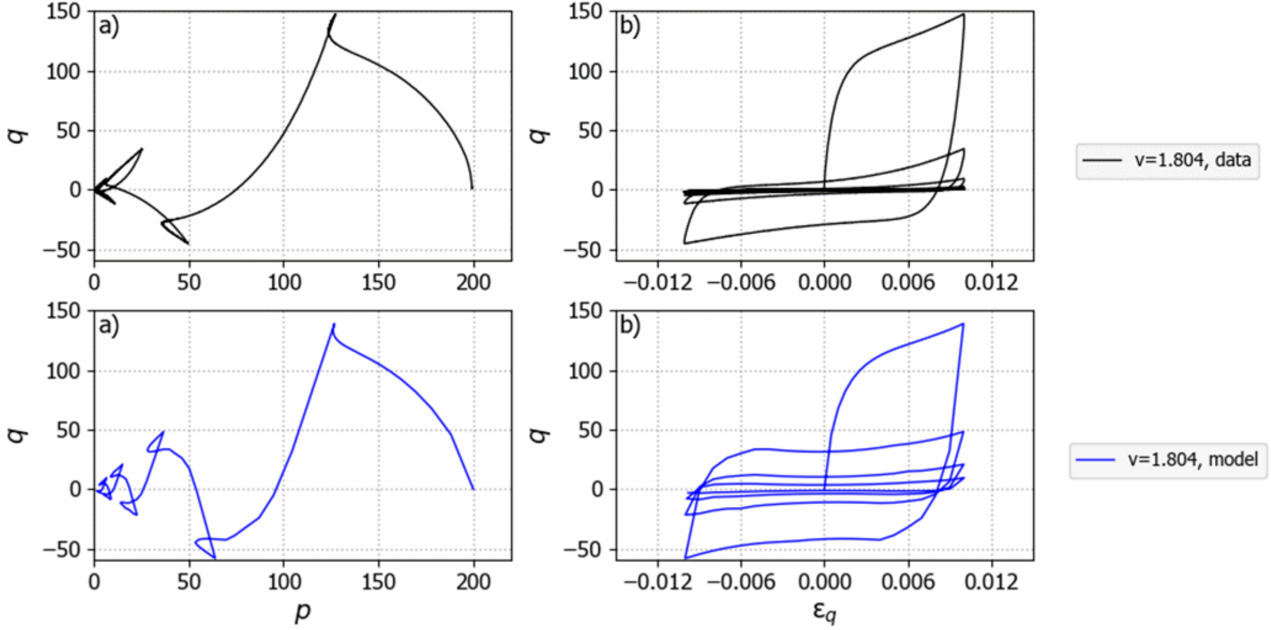
Cyclic tests: strain controlled (large cycles)

Data

Model



Data by Wichmann,modelling by Simonin

… final remarks

- Modelling cyclic loading
- Hyperplasticity ensures that the models obey thermodynamic principles
- HARM is a method to add ratcheting to basic multisurface models
  - Reproduces “contour diagrams”
- HySand models cycling of sand
  - Reproduces Wichtmann’s data well